TARGET ALLOCATION FOR FIELD ARTILLERY

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THESIS

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ABSTRACT

Several models of the problem of target selection for field artillery fire as a supporting weapon system to a maneuver element in a division field environment are presented in this thesis. The field artillery system, its capabilities and limitations, as well as, the criteria utilized by military decision makers to provide timely, accurate, and effective artillery fire support to the maneuver commander, is covered to familiarize the analyst with the system to be modeled. A differential equation model using Lanchester Theory of Combat and the mathematical technique of optimal control to the target allocation problem is presented. A second model presented uses an allocation of fire dependent upon the kill potential and capability of the respective forces. The kill potential varies with the lethality and range of the weapon system from that force. A discussion of the worth of combat units in dynamic combat situations is also presented. The conclusion reached is that there is a dire need for more models in the area of target allocation that can clearly depict reality and still maintain a certain mathematical tractability.

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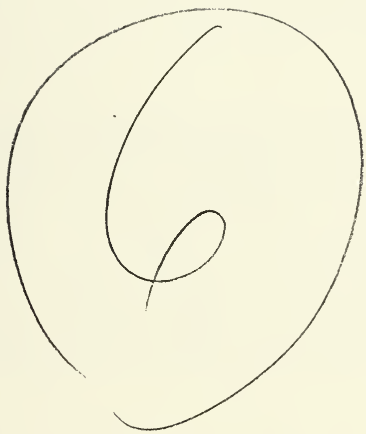


I. INTRODUCTION

The purpose of this thesis is to develop a simplified mathematical model to gain insight into the best target selection for field artillery fire as a supporting weapon system to the division maneuver element. The concentration of effort will be to develop theoretical models to present the engagement of combat.

The value of theoretical models to depict any system can be best expressed by a quote from Clausewitz in his study on the book, "On War":

> "If theory investigates the things that make up war, if it separates more distinctly that which at first seems confused, if it explains fully the properties of the means, if it shows their probable effects, if it clearly defines the nature of the ends in view, if it sheds the light of a deliberate critical observation over the whole field of war then it has achieved the main object of its task. It then becomes a guide to whoever wishes to become familiar with war from books; it everywhere lights up for him the road, facilitates his progress, educates his judgment, and keeps him from going astray." [6]

This philosophy on models and theory is as appropriate today as it was in the 1800's when Clausewitz said it.

The first attempt at the development of combat models was in the form of simultaneous differential equations formulated by F. W. Lanchester. Since this development much work has been accomplished in extending these Lanchester equations by relaxing one or more of the assumptions. Among some of these extensions there has been considerable effort devoted to the area of differential games and control theory depicting

primary maneuver systems supported by secondary weapon systems. It is from these latter extensions that the author will model the artillery allocation problem.

In the presentation of any modeling situation the analyst is faced with the well known analysts' dilemma. On one side, the model developed can be so close to reality that the model becomes so detailed and complex that it is not tractable and therefore yields no meaningful information. On the opposite side of the picture, the model can be so simplified by assumptions that the real world situation attempted to be modeled becomes too far removed from the end product. It is somewhere in the middle-ground that is the goal of all analysts, and this will be the case in the modeling of the artillery problem.

To successfully model any system it takes a great deal of experience and a thorough knowledge of the system to be modeled. In conjunction with this idea a detailed discussion of the inner functions of the artillery system is presented to the reader as background material.

It is then with this introduction and the desire to add to the ever increasing models being developed of combat that this paper is written.

II. THE FIELD ARTILLERY SYSTEM

A preliminary requirement in the formulation of any model is a thorough knowledge of the system that the analyst is attempting to model. A thorough understanding of the field artillery system requires knowledge of its organizations, the roles or missions that the artillery is called upon to perform, its capabilities and limitations, the criteria used by military decision makers to provide adequate artillery fire to the maneuver commanders, and the environment in which the artillery conducts operations.

A. ROLE OF THE FIELD ARTILLERY SYSTEM

The general mission of the field artillery in combat is to:

1. Provide timely, close, and accurate fire support to the maneuver elements, destroying or neutralizing those targets that are most likely to jeopardize the accomplishment of the mission.

2. Add depth to the battlefield by attacking reserves, command posts, logistical installations, and lines of communications.

3. Achieve fire superiority over enemy mortars, artillery, and nuclear delivery systems within its area of coverage and responsibility.

To accomplish the above listed general mission requirements the field artillery system must call upon all of those

elements necessary to obtain the desired effect on the target complex. These elements that make valuable contributions to yield the desired results on the target as listed in Ref. 1 include:

The firing weapon or firing units
Target acquisition element
Survey element
Ballistic metrological element
Communications element
Logistical element
Fire control and coordination element
Transportation units

B. THE FIELD ARTILLERY TEAM

The elements of the artillery system that the analyst is most concerned about for combat modeling are the firing unit or the weapon, which is referred to as the firing battery; the target acquisition element, which is more often thought of as the forward observer; and the fire control element, which is denoted as the fire direction center. These are the primary elements of the artillery team, although to successfully accomplish the overall mission, the other five elements mentioned above contribute their needed support, to aid the artillery team to place the "steel" on the target.

The functions of the primary elements of the "artillery team" are:

1. Fire Direction and Control

The fire direction center analyzes incoming information from its target acquisition sources, especially its

forward observers, determines the necessary firing data for the firing units, and transmits this firing data via the existing communications to the firing units. The units to fire are also determined by the fire direction center.

2. Target Acquisition

The forward observer detects enemy targets in his area of responsibility, reports these to the fire direction center, initiates fire missions, and adjusts fire as required. Although the forward observer is the eyes of the field artillery team, there are other means of target acquisition such as, countermortar radars, sound devices, night vision devices, captured enemy documents, and the like that perform an equally important role as target acquisition for the artillery.

3. Firing Units

The firing units at platoon, battery, or battalion level, apply the firing data transmitted by the fire direction center to the artillery pieces to fire and actually execute the fire mission.

To successfully attack any target complex to the requirements of the field commander it requires a concentrated effort of all the members of this field artillery team.

C. TACTICAL MISSIONS OF THE FIELD ARTILLERY

To yield flexibility and responsiveness to the field commanders desires the field artillery has available four standard tactical missions it can assign to its battalions. Modifications to these four standard missions are possible,

but these are more generally used only when the intent of the commander cannot be clearly specified by one of the four basic missions. The standard missions designate necessary fire support responsibilities among supported and supporting units in the area of communications, liaison, zones of fire, observer requirements, responsibility for unit displacement, response to calls for fire, and responsibilities for planning of artillery fires. These four missions are listed in Appendix D, in descending order as to the amount of centralized control retained by the commander; general support, general support-reinforcing, reinforcing, and direct support [Ref. 7].

Although these tactical missions designate necessary control for the artillery, there exists a basic requirements list inherent to each of these assigned tactical missions that is of considerable importance to the analyst who is attempting to develop meaningful, tractable models of the artillery system. These are best identified by two broad categories, offensive and defensive operations [7].

1. Offensive Operations

Some of the missions that the artillery is required to perform during offensive operations are:

a. Provide adequate support for the plan of maneuver of the supported maneuver force.

b. Utilize the weapons within their listed capabilities.

c. Furnish massed fires throughout the zone of action of the supported maneuver force.

d. Facilitate future tactical operations.

e. Provide necessary harassing and interdiction fire.

2. Defensive Operations

The more common missions that the field artillery is called upon to execute during defensive operations are:

a. Provide necessary harassing and interdiction fires to force the enemy into early deployment and to disorganize and disrupt his operations.

b. Delay and disorganize the enemy's approach.

c. Disrupt the enemy's attack preparation by effective counter-preparation fire.

d. Impede the enemy's attack with close defensive fires in width and depth throughout the zone.

e. Break up the enemy's assault with final protective fires and barrages.

f. Limit penetration with on call fires within, behind, and forward of the forward edge of the battle area (FEBA).

g. Support the counterattack and associated offensive actions of our force.

These artillery requirements are general in nature. To accomplish these listed offensive and defensive operations requirements there must exist a truly effective, artillery team that is responsive to the commander's wishes providing

timely, effective, accurate and continuous supporting fires. This continuous fire is contingent upon continuous fire planning of artillery fires on targets of opportunity and planned targets.

Targets of opportunity are targets which are observed after the beginning of an operation, whereas planned fires are preplanned on areas or points for which fire may be needed to accomplish the overall tactical mission before the operation begins. These planned fires include a long list of types of fires such as, scheduled fires, on call fires, concentrations, barrages, preparations and interdiction fire to name only a few.

Although planned fires allow normally for sufficient analysis to properly allocate units and ammunition, effective fire on targets of opportunity requires proper employment of intelligence agencies, effective interchange of information at all levels of command, rapid employment of the most effective weapon system to engage the target, sufficient ammunition on hand, proper and efficient action on the part of the artillery team and close and continuing coordination with all adjacent and concerned tactical units [8].

D. LIMITATIONS AND CAPABILITIES OF THE FIELD ARTILLERY

In accomplishing these varied missions associated with the field artillery the analyst must also be aware of the existing limitations and capabilities of the artillery [7].

1. Capabilities

The artillery has the capability to:

a. Shift fires rapidly within a large area and on a wide front without displacement.

b. Mass fires on one or more targets.

c. Place indirect fire on targets in defilade positions.

d. Deliver accurate fire with proper ammunition under adverse conditions from weapons emplaced laterally and in depth throughout the zone of action.

e. Place fire on targets from a defilade position.

f. Deliver accurate fires on targets without adjustment.

g. Displace rapidly to new positions and employ artillery units to mass fires when required.

h. Conduct assault fire to effect destruction of point targets.

i. Conduct direct fire against assaulting troops.

j. Provide aerial artillery fire support.

2. Limitations

The limitations restricting artillery fire are:

a. Mission effectiveness is considerably reduced and vulnerability is increased during displacements.

b. Mission effectiveness is considerably reduced when it is required to engage in close combat with the enemy.

c. Mission effectiveness is reduced during enemy air attack.

E. ARTILLERY UNITS AVAILABLE AND THEIR CAPABILITIES

To discuss target allocation models in the artillery, it is essential that the analyst have a thorough knowledge of the firing units available and their listed capabilities, as well as their method of employment.

Artillery is employed in great width and depth throughout the battle area to provide adequate and overall coverage of the tactical zone. Artillery is classified as light, medium, heavy, or very heavy [9].

Type	Criteria for Classification
light	120mm or less
medium	greater than 120mm, but less than 160mm
heavy	greater than 160mm, but less than 210mm
very heavy	greater than 210mm

The exact positioning of artillery units so as to effectively influence the outcome of the battle is dependent on such factors as terrain, combat situation, actions of supported unit, tactical mission, and other related factors.

The maximum range of artillery weapon systems most commonly employed to support maneuver forces directly are listed below from Ref. 9 to provide an idea of artillery capabilities.

Weapon	Maximum Range (meters)
105mm(towed or SP),M108	11,500
155mm(towed or SP),M109	14,600
8 inch SP, M110	16,800
175mm gun, M107	32,700
*4.2 inch mortar	5,650
115mm multiple launcher,M91	10,600

* - Located with the infantry organization(organic).

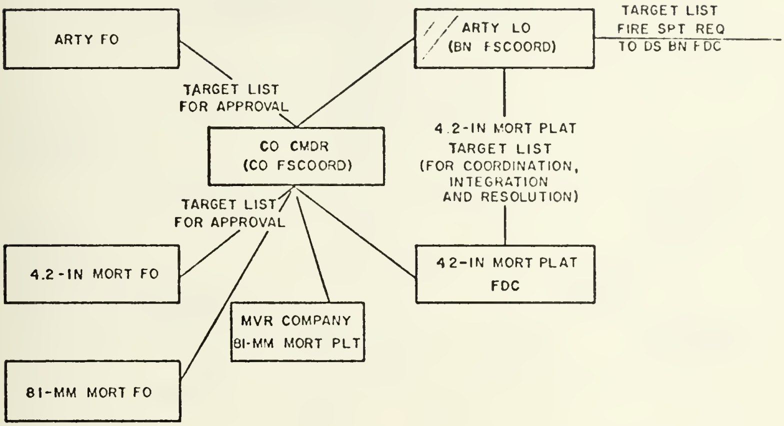
These artillery type units normally located at Division, Corps, or Army level are capable of supporting operations in varying type terrains and climates. These variations include the flat, hot and dry desert; the humid and wet areas of swamp land and jungles; as well as the high climates of the mountain country. Performance under various degrees of mobility to include mechanized or non-mechanized operations, water or canal operations as in Southeast Asia, and air-mobile operations with the smaller and lighter weapons is another of the assets of the field artillery. Further the artillery is flexible to fight under any environment or situation, and it is therefore important that the analyst be specific in his model formulation to state what the situation is and what the assumptions are for each model.

F. ARTILLERY FIRE PLANNING

To provide immediate, responsive, timely, accurate and effective artillery fire support to the field commander of the maneuver elements the artillery must conduct continuous fire planning. This fire planning must be effective at all levels of command from the battery size unit up to Army level.

Fire planning is accomplished simultaneously at all levels with a current updating of information as often as is possible or necessary. Figures 1 and 2 depict the fire planning channels for company, brigade, division artillery, and corps artillery levels. To familiarize the reader with what normally transpires in a fire planning process the discussion will concentrate on the brigade and division artillery levels.

FIRE PLANNING CHANNELS
(COMPANY LEVEL)



FIRE PLANNING CHANNELS
(BRIGADE LEVEL)

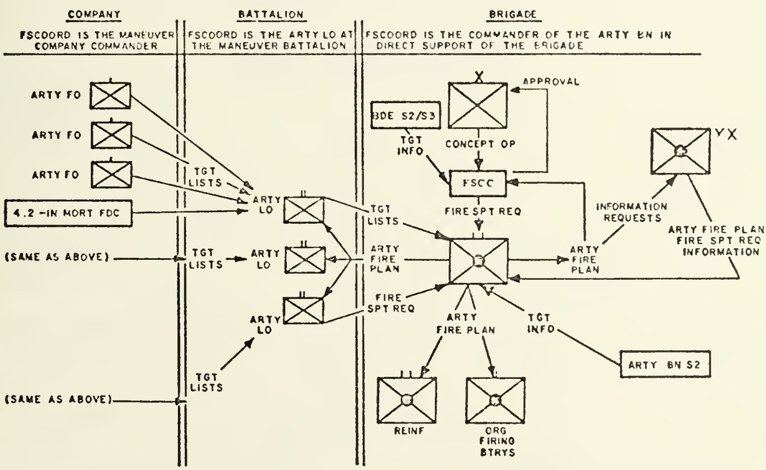


Figure 1. Fire Planning Channels Company and Brigade Levels

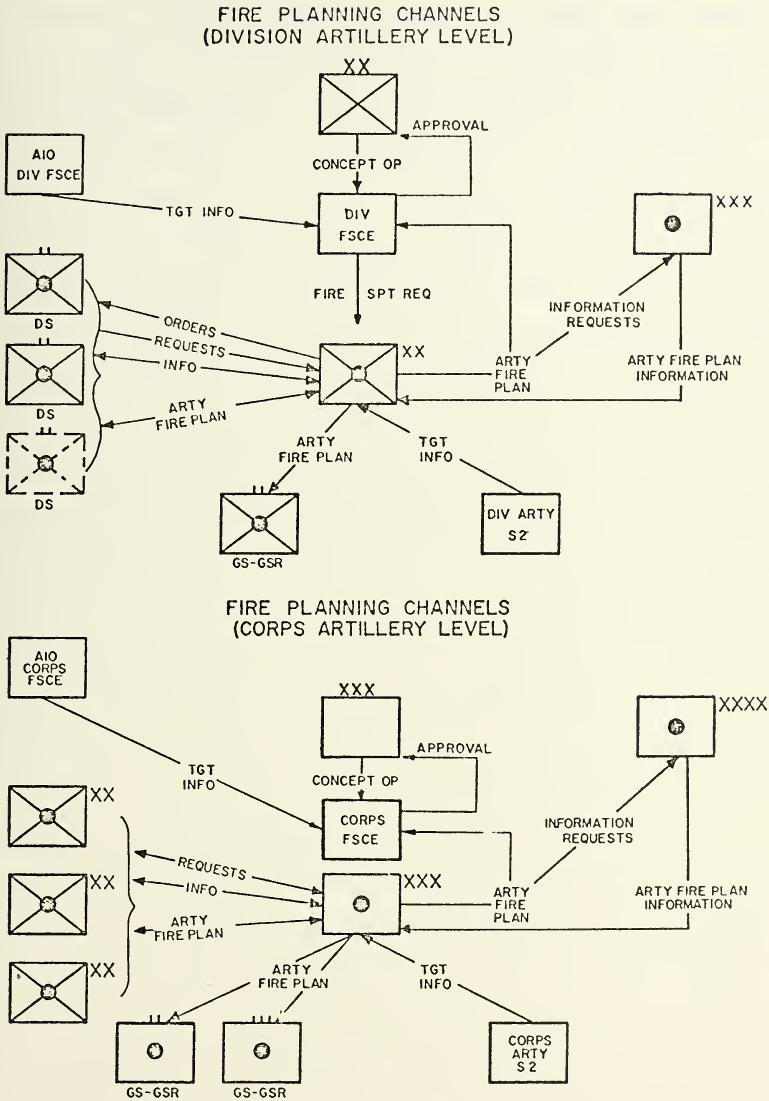


Figure 2. Fire Planning Channels - Division and Corps Artillery Levels

In situations involving the direct support battalions in support of an infantry brigade continuous fire planning is the key. As is shown in Figure 1 the direct support battalion artillery S-3, operations officer, receives detailed fire plans and requests from each of his liaison officers located with their supported infantry battalions. These liaison officers in turn compiled their plans from fire requests from the supported infantry units along with planned targets in the area of responsibility of each of the forward observers located with each of the three infantry line companies in the battalion. The forward observer located with the infantry company is the senior artillery advisor to the small unit company commander advising him on the availability of field artillery fire to best accomplish his mission. The liaison officer likewise advises and recommends to the infantry battalion commanders the best way to employ or use the artillery fire available to support the battalion mission. In addition, the liaison officer collects and consolidates the target lists from each of his forward observers for forwarding to the artillery battalion operations officer, S-3. Then the S-3 takes all the liaison officers fire plans and consolidated requests along with target information generated from the direct support artillery battalion intelligence section, the brigade liaison officer, adjacent units, and other intelligence agencies available to him and he compiles a typical artillery fire plan. To construct an artillery fire plan the operations officer must accomplish the following:

1. Plot the fire units and their ranges and traverse limits on an overlay to be affixed to the planning map.

2. Plot targets and designate other targets appropriate.

3. Resolve any duplication of fires.

4. List the targets received from the planning sources on the target list.

5. Select the targets to be scheduled and the method of attack, using the appropriate columns of the target list.

6. Prepare an artillery fire plan table for those targets to be fired in accordance with a time schedule; for example, preparations fires, series of targets, and schedule of fires.

7. Prepare the written portion of the fire plan.

These general procedures in preparation of an artillery fire plan are accomplished basically the same way at all artillery levels. Reference 10 illustrates a completed fire plan with all its annexes, appendices, and tabs.

Once the S-3 is satisfied that the fire plan will best accomplish the fire mission of the field commander, he submits the fire plan for the brigade commander's signature and approval while at the same time he forwards a copy of the plan to the division artillery for information. Upon completion of this activity with the approval of the commander the fire plan is published and submitted to all units concerned for implementation.

At the division level the division artillery S-3 prepares the artillery fire plan to support the division, based on the division commander's concept of the operation. The objective

of the division artillery planning is coordinated artillery fire support for the entire division area. A division artillery fire plan is initiated by planning fires for the division general support artillery and by planning fires on targets requested by, or beyond the range of the direct support battalions, and by planning fire on targets of interest to the division zone as a whole. Fire plans of the direct support artillery are augmented when necessary by general support and reinforcing fires, which are integrated with the fire plan of the division artillery. Targets which are incapable of limited interference with the division level plan of maneuver are sent to the S-2, intelligence section, of the direct support artillery battalion for consideration for attack on the target at that level. Targets out of range of division artillery are sent to the next higher headquarters, Corps artillery. Targets unsuitable for attack by artillery are sent to the division fire support coordination element for possible attack by other fire support means, such as, tactical air, naval gunfire, or other means. All counter-battery type targets are sent to division and to corps artillery for attack consideration. It is only at these two levels that counter-battery programs can be coordinated for subordinate units.

It should be obvious that fire planning is conducted concurrently at all levels of artillery fire planning. It does not cease when a formal fire plan is published and disseminated. Additions, deletions, and necessary updating must be transmitted to appropriate headquarters when the need arises or as changes occur.

All targets or potential targets are examined and analyzed to determine their military importance, priority of attack, and capabilities for available weapons to attack. The length of time and the amount of detail in making a target analysis depends on the amount of information available concerning the target, the means available for attack, the degree of necessary coordination required, and the urgency for attack. Planned targets receive a more detailed analysis than targets of opportunity. Even though targets of opportunity receives only a cursory mental calculation in order that timely, accurate, and effective fire may be brought to bear on the fleeting target, the same considerations run through the mind of the operations officer as were considered for the planned target.

The basic consideration in making a target analysis is the concept of the operation as announced by the commander. Firepower must be integrated properly with the scheme of maneuver. Standard operating procedures in the units, as well as local commander's policies must be adhered to by the target analyst in his analysis.

The military importance of a target is determined by the threat the target represents. The importance of the target is valid only at the level of force at which the analysis is conducted. A guide for priorities in ascertaining military importance as extracted from Ref. 8 is listed below:

Priority	Target's Capability
I	Prevent the execution of the plan of action
II	Immediate interference with plan of action
III	Ultimate interference with plan of action
IV	Limited interference with plan of action

In addition to these listed priorities there exists a list of criteria for precedence of fires, since rarely is there sufficient fire support available to undertake fire on all targets. Characteristics to be considered for target engagement are as extracted from Ref. 8 indicated below:

Nature of target - Composition, size, shape, vulnerability, mobility, and recuperability.

Location - Closeness to friendly troops, proximity to area commanders desires to damage.

Terrain and Weather - Vulnerability of target. Visibility of target and effect of attack by fire support means.

Desired Effect - Neutralization, destruction or harassment.

Once the priorities and precedence are established the weapon system to produce the desired effect must be chosen. Considerations here include desired effect, troop safety, and time requirements.

Subsequently the desired method of attack is established. The desired method of attack is based on the mean point of impact determined by the target size and shape, the density of fire, the duration of fire which is dictated by the ammunition available and the desired effect, and if surprise or non-surprised fire is desired since fire effectiveness can be improved if fire without adjustment is used.

Finally after fire is delivered the final step is target assessment. This can be accomplished by information sources such as target acquisition agencies that identified the target initially, imagery interpretation, inspection of the area after capture, prisoner reports, or captured documents. Whichever source provides the needed results in the particular situation is the one chosen.

Pertinent data from the assessment reports not only give the S-3 immediate indications where to change the impact of fire, but in addition when properly analyzed the assessment can determine comparative effectiveness of artillery weapons, techniques, and ammunition effects in the attack of target complexes of varied hardness.

G. GENERAL COMMENTS

It has been previously stated and emphasized several times that artillery fire must be timely, accurate, and effective. This suggests to both the artilleryman and the analyst that there must exist a sense of urgency on the artillery team. Artillery fire that is late is just as ineffective as artillery fire that was inaccurate or not delivered at all. Therefore every element of the artillery team must be totally responsive to the supported maneuver elements requirements.

In a world of fast moving, complex technology and increased mobility, artillery fire to be effective must provide adequate fire in both volume and proper type to provide the desired results in the target area. A failure to accomplish this will result in a failure to accomplish the mission and a loss of the objective.

Finally, if the fire is timely accurate and effective, but there does not exist sufficient or adequate detection devices for target acquisition, again the artillery will fail to provide the commander with needed supporting fires.

With the ultimate objective of the artillery in mind, that is to deliver accurate, timely, and effective firepower in support of the maneuver elements in order to inflict a maximum of casualties and to destroy the enemies will be to fight, it is then the purpose of this thesis to hopefully contribute or to extend the already growing volume of models depicting the problem of target allocation of artillery fire in support of the maneuver commander.

III. TACTICAL SITUATION

As it was previously emphasized, model formulation of a real world problem requires the analyst to specify the circumstances and conditions under which the model is to be considered. It is the object then to list the conditions for modeling the target allocation of field artillery fire by describing the tactical situation in the division area.

A. LEVEL OF COMBAT

According to military tacticians and strategists there exists three main levels of combat and these are low, medium, and high intensity warfare. Low intensity warfare is considered to be equivalent to the type of warfare which is presently being engaged in or conducted in Southeast Asia. Medium intensity warfare is of the caliber conducted in the past two world wars and in Korea. Finally, all out nuclear warfare is classified as high intensity warfare. Most discussions by military planners and analysts are generated around tactics and strategy involving medium intensity warfare, and such will be the case for the discussion presented here.

B. UNIT SIZE IN MODEL

In considering medium intensity conflict in discussing the target allocation problem, several levels of tactical areas can be presented ranging from the squad to field army levels. It is strongly felt that the field artillery

allocation problem could best be exemplified at the division level where the salient points could readily be depicted for analysis. A typical infantry division will be presented for both the friendly (X - force) and the enemy (Y - force) forces composed basically of infantry as the primary force and the artillery as the secondary or supporting force. Although the infantry division is composed of forces of other types; such as, armor, engineers, and necessary logistical support elements, to avoid detracting from the problem at hand these other divisional elements will not be considered, but they will be left as a further area of extension for other analysts so inclined. In addition, other supporting secondary forces as naval gunfire, tactical air, and nuclear means will likewise be disregarded for the above mentioned reasons.

C. WEAPON SYSTEMS AVAILABLE

Weapon systems available for the division forces will include a direct fire weapon (could be considered an M-14 or of similar caliber) with known characteristics and capabilities. Weapons for the X and Y forces will be different and this will be characterized by the kill coefficients used in the model. Although it would not be an insurmountable problem to consider elements of the infantry forces armed with different weapons, it would be uneconomical to do so in this paper.

The secondary system or the artillery in the division will be typical of a normal infantry division artillery, in that it will be composed of three (3) - 155mm artillery battalions,

each with three (3) firing batteries of 6 howitzers; and one (1) 155-8 inch battalion, composed of three (3) batteries of 155mm with six (6) howitzers each, and one (1) 8 inch battery of four (4) guns. In addition, to signify other artillery, non-divisional, 8 inch battalions composed of three (3) batteries of 4 guns each, will be in general support. The opposing artillery will be made up of a similar structure but with different weapons of different characteristics.

Each of the above mentioned units will have the range characteristics mentioned in Section II, E. Opposing force artillery ranges although not listed will be considered of equal quality.

NOTE: It should be realized that targets normally attacked by other forces to include higher caliber artillery not found at division level or tactical air, are to be considered suitably attacked and annihilated and therefore exogenous to the situation and therefore exogenous to the model.

D. ARTILLERY ASSIGNED MISSIONS

Typical missions for divisional and non-divisional artillery units are as shown in Appendix D. Specifically for the 155mm units the mission of direct support is established and for the 155/ 8 inch composite battalion and the non-divisional battalions a mission of general support is assigned. These assignments are those normally established, and the associated responsibilities are also listed in Appendix D.

E. CURRENT FORCE STRUCTURE AND FORCE DISPOSITION

1. Current Force Structure

Forces for both sides will be structured with three (3) infantry brigades designated X_i and Y_i for the friendly and enemy forces respectively, where

X_1^j - represents the friendly brigade where $j = 1,2,3$ signifying the 1st, 2d, and 3d brigades respectively.

X_2^k - represents the friendly supporting artillery where $k = 1,2,3$ signifying 155mm, 155/8 inch, 8 inch battalions respectively.

Y_1^j - represents the opposing brigades where $j = 1,2,3$ signifying the 1st, 2d, and 3rd brigades respectively.

Y_2^k - represents the opposing artillery where $k = 1,2,3$ signifying similar caliber artillery of comparable characteristics.

2. Disposition of Forces

A division area is normally constituted with two brigades up front and one in reserve. This tactical deployment is utilized down to and including squad level; that is, two elements up one in reserve. The configuration of this type tactic is depicted in Figs. 3, 4, and 5.

The model discussion will concentrate on Fig. listing the division area with the smallest infantry unit considered to be a brigade and the smallest artillery unit will be the supporting field artillery unit, the battalion.

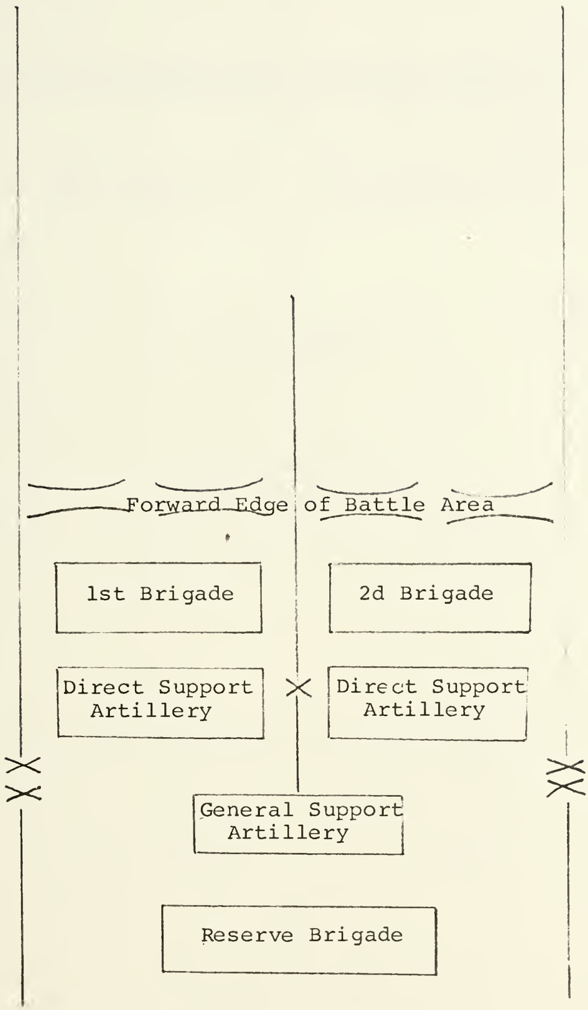


Figure 3. Division Area (2 Brigades up and 1 back).

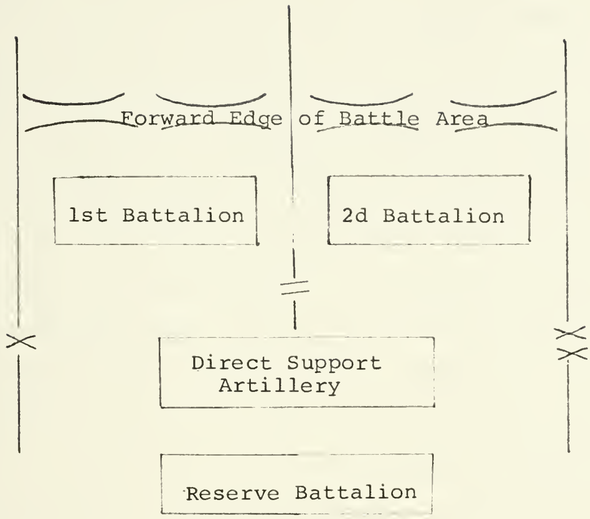


Figure 4. Brigade Area (2 Battalions up and 1 Back).

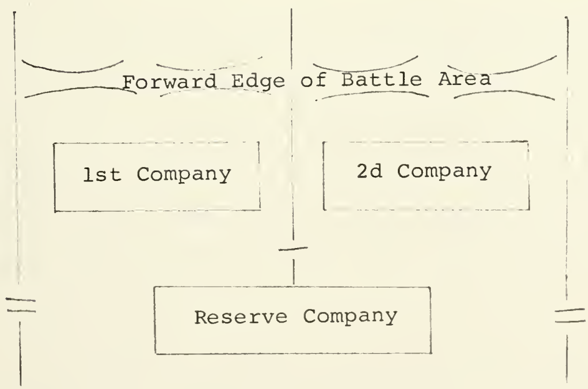


*Figure 5. Battalion Area (2 Companies up and 1 Back).

* Notice Artillery supports brigades, not battalions.

For this disposition of forces the 155mm artillery battalions of the division will be assigned missions as follows: the two artillery battalions supporting the two brigades on line will be direct support on order general support should their respective brigade be placed in reserve; the other 155mm artillery battalion normally in direct support of its brigade will be general support on order direct support to its brigade when it is committed in the battle, and the 155mm/8 inch battalion and the non-divisional artillery will be general support of the division operations. The mission of general support reinforcing is also a frequent choice in situations like this, but general support gives the commander more centralized control as was indicated in Section II, C.

Individual batteries on some isolated occasions may receive separate missions, but this is the exception rather than the rule.

The exact positioning of the artillery forces in support of the division is such as to follow the artillery rule of thumb of locating battery units such that two-thirds of the range of the unit is extended in front of the forward edge of the battle area (FEBA), and to facilitate coverage of the entire division area while a sufficient amount of overlapping fires exists.

The infantry brigades are positioned with a figure of 1000 meters per company for defense and adjacent units

with interlocking fire. This figure of 1000 meters is flexible to adjust to existing terrain configuration.

F. TERRAIN LIMITATIONS

The terrain for the problem is considered to be normal rolling flat terrain and standard vegetation with no hindrance to troop movement or indirect fire support for either force. Forces on both sides make use of existing cover and concealment and are so deployed to the best utilization of existing terrain.

G. GENERAL

This then is the situation for which the models of field artillery allocation will depict. It should be noted that the number of infantry brigades or supporting artillery battalions or types could easily be altered by simply extending the use of subscripts or superscripts to facilitate one's purpose. In addition, all the division's forces could be included with kill coefficients compatible to the type of weapon system the force utilized, but for this case it would add unnecessary cluttering and difficulty to the discussion of the problem to be attacked. The reason then to limit the type of forces for discussion and to list specific type units is to facilitate the problem discussion and to facilitate ease of model presentation.

IV. MODELING THE SYSTEM

A. BACKGROUND OF SOME THEORETICAL MODELS

The object now is to model this tactical situation presented in Section III to yield some useful information on the target allocation problem in the field artillery. Some theoretical models are already in existence that have been found to be consistent with established facts of warfare. It is from these existing theoretical models developed by notable analysts that the author will try to model this artillery allocation problem. With the idea of extending already developed theoretical models, it is then the purpose of this section to present a background of some of these basic theoretical models of combat.

The first attempt at the development of combat models was by F. W. Lanchester in the form of differential equations [11]. This formulation of differential equations by Lanchester is commonly referred to in the literature as Lanchester equations. A brief review of Lanchester's development of the well known Linear and Square Laws and their solution is presented in Appendix A.

Since the development of the Lanchester Equations many valuable contributions to the field have been accomplished by extending the equations simply by relaxing one or more of the assumptions used by Lanchester. A recent comprehensive paper by Seth Bonder [19] has summarized the works of analysts responsible for these extensions. In Ref. 20 a particularly

noteworthy contribution is presented by Herbert K. Weiss, in his noteworthy treatment of relative movement of forces, combat between small forces in the presence of weapons with large areas of effectiveness and finally combat among heterogeneous forces. The problem of target assignment is covered when the forces are heterogeneous.

Weiss, in Ref. 21, used Lanchester equations for a combat model in a problem which he formulated as a differential game. The tactic that Weiss has analyzed is the selection of targets to be attacked. This tactic is incorporated in his model which includes two forces in contact where each force consists of a primary and a secondary system. The two system game could be infantry and the supporting artillery. A brief review of this game and Weiss's solution is presented in Appendix B.

In Ref. 5 Brackney extended Lanchester equations to include the concept of search during combat for the individual combatants of the opposing force. This development is made with the assumption that the distribution of a combat force, over an area held by that force, is uniformly random. Lanchester's assumptions for his square law are also present. A summary of Brackney's work is not presented here, but it is available to the interested reader in Ref. 5.

Further, in Ref. 15, Col. Thomas S. Schreiber developed a model using differential equations, which enables the efficiency of the intelligence and command and control systems to quantitatively related to the fire power and numerical

strength of a force. He presents numerical results which indicate that an increase in the target acquisition capability can be equivalent to an increase in force strength. This implies that the combat effectiveness of a unit can be increased significantly by improving the unit's target acquisition capability.

More recent extensions of combat theory are presented by J. Taylor in Refs. 16 and 17 in the area of target selection for combat between heterogeneous forces. He has examined several simplified idealized combat situations for the purpose of interpretation to yield general principles to serve as stepping stones for further studies in field experimentation and model simulation.

B. MODEL 1

With this background of theoretical models and ideas of combat using Lanchester Theory and other analysts extensions in related areas to the theory, it is then the purpose to model the problem of target allocation of field artillery in a supporting role of an infantry element in the stated division tactical situation. The forces of both friendly and enemy sides are both static with neither side in an attack posture as in Fig. 6.

Assumptions for this initial model are indicated below:

1. Target acquisition rate is greater than firing rate.
2. Unlimited ammunition available.
3. Perfect information of opposing forces.

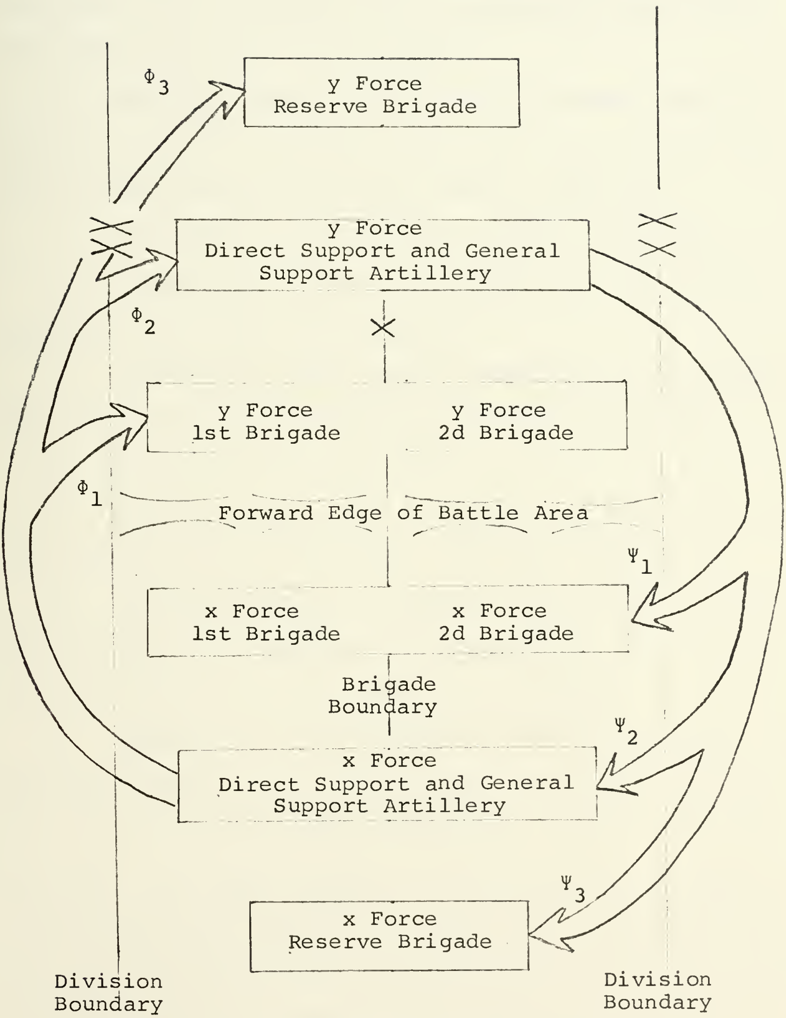


Figure 6. Tactical Force Disposition

4. Ground forces in primary force composed of infantry elements (armor, etc., not considered to aide in problem simplification).

5. Allocation of fires by operations officer, S-3, is considered to be rational and in conjunction with listed artillery priorities and procedures.

6. Artillery weapons do not have to adjust fire on targets, they are considered registered and they can shift fires inside the division area without limitation.

7. Replacement rate of troops is not considered.

8. No operational losses to either side like accidents, dysentery, and AWOL's are included.

9. Infantry can only inflict casualties on infantry; while the artillery can inflict casualties on either.

10. Problem is time specified.

11. Infantry fires aimed fire and obeys square law; artillery fires area fire and obeys linear law conditions.

12. Infantry elements in reserve cannot inflict casualties until committed since they are out of range, but they do possess a kill coefficient and kill potential.

13. Targets normally attacked by other forces other than those considered, for example higher artillery or tactical air, are so attacked and considered exogenous to the model.

Certainly with these assumptions and with the knowledge of already developed theoretical models, this situation would fit nicely into the theory developed by H. K. Weiss, listed in Appendix B. with extensions as applied by J. Taylor in

Ref. 16. Using this approach and noting that assumption 9 alters the state equations to include casualties imposed by the infantry, the problem can be formulated as a time specified problem, where the objective for the X - force is to minimize losses while maximizing the oppositions losses. The problem is a one-sided version problem evaluated for specified Ψ's for the Y - force to determine the Φ's for the X - force.

The payoff to X in the stated objective function is

$$\text{minimize} \quad p[X^1(T) + X^2(T) + X^3(T)] + q[X^1(T) + X^2(T) + X^3(T)] -$$

$$(1) \qquad r[Y^1(T) + Y^2(T) + Y^3(T)] + s[Y^1(T) + Y^2(T) + Y^3(T)]$$

subject to the state equations

$$(2) \quad \dot{x}_1 = \frac{dx_1}{dt} = - a_1 y_1 - a_2 \Psi_1 x_1 y_2$$

$$(3) \quad \dot{x}_2 = \frac{dx_2}{dt} = - a_2 \Psi_2 x_2 y_2$$

$$(4) \quad \dot{x}_3 = \frac{dx_3}{dt} = - a_2 \Psi_3 x_3 y_2$$

$$(5) \quad \dot{y}_1 = \frac{dy_1}{dt} = - b_1 x_1 - b_2 \Phi_1 y_1 x_2$$

$$(6) \quad \dot{y}_2 = \frac{dy_2}{dt} = - b_2 \Phi_1 x_2 y_2$$

$$(7) \quad \dot{y}_3 = \frac{dy_3}{dt} = - b_2 \Phi_3 y_3 x_2$$

where $x_1 = x_1^1(T) + x_1^2(T)$, front line brigades

$x_2 = x_2^1(T) + x_2^2(T) + x_2^3(T)$, artillery fire support units

$x_3 = x_1^3(T)$, reserve force brigade

with the same notation for the opposing forces but using y's.

a_1, a_2 - represent the kill rate coefficients of the y infantry and artillery forces.

b_1, b_2 - represent the kill rate coefficients of the x infantry and artillery forces.

$p, q, -r, -s$ - represent the weighting factors assigned to the surviving forces by the x commander.

Φ_i and Ψ_i - represent the proportion of artillery fire allocated for the x force and y force directed at the other force.

Ψ_i are specified, and $0 \leq \Psi_i, \Phi_i \leq 1$

where $\Psi_3 = (1-\Psi_1-\Psi_2)$ and $\Phi_3 = (1-\Phi_1-\Phi_2)$

since a prescribed duration time T is being used then

$$
\begin{array}{lll}
p_1(t=T) = p & & x_1 = x_1^0 \\
p_2(t=T) = q & & x_2 = x_2^0 \\
p_3(t=T) = p & \text{and at time} = 0, & x_3 = x_3^0 \\
p_4(t=T) = -r & & y_1 = y_1^0 \\
p_5(t=T) = -s & & y_2 = y_2^0 \\
p_6(t=T) = -r & & y_3 = y_3^0
\end{array}
$$

The Hamiltonian is

$$(8) \quad H(x,y,p,t;\Psi,\Phi) = -p_1a_2\Psi_1x_1y_2 - p_2a_2\Psi_2x_2y_2 - p_3a_2\Psi_1x_3y_2 - p_4b_2\Phi_1x_2y_1 - p_5b_2\Phi_2x_2y_2 - p_6b_2\Phi_3x_2y_3 - p_1a_1y_1 - p_4b_1x_1$$

and the adjoint equations are

$$(9) \quad \dot{p}_1 = -\frac{\partial H}{\partial x_1} = p_1 a_2 y_2 \Psi_1 - p_4 b_1$$

$$(10) \quad \dot{p}_2 = -\frac{\partial H}{\partial x_2} = p_2 a_2 y_2 \Psi_2 + p_4 b_2 y_1 \Phi_1 + p_5 b_2 y_2 \Phi_2 + p_6 b_2 y_3 \Phi_3$$

$$(11) \quad \dot{p}_3 = -\frac{\partial H}{\partial x_3} = p_3 a_2 y_2 \Psi_3$$

$$(12) \quad \dot{p}_4 = -\frac{\partial H}{\partial y_1} = p_4 b_2 x_2 \Phi_1 + p_1 a_1$$

$$(13) \quad \dot{p}_5 = -\frac{\partial H}{\partial y_2} = p_1 a_2 x_1 \Psi_1 + p_2 a_2 x_2 \Psi_2 + p_3 a_2 x_3 \Psi_3 + p_5 b_2 x_2 \Phi_2$$

$$(14) \quad \dot{p}_6 = -\frac{\partial H}{\partial y_3} = p_6 b_2 x_2 \Phi_3$$

Since the adjoint equations now depend on the state variables, the resulting two boundary value problem does not readily possess a solution obtainable by elementary methods. However, the above model is believed to be a realistic model for deployment of the artillery supporting system against ground forces, since the individual soldiers are not engaged as point targets [16]. As Taylor states in Ref. 16 in his discussion of a second extension from Weiss [21], that in cases of partial information that the supporting unit is informed about the general areas in which the opposing infantry are located, but is not informed about the consequences of its own fire. This still applies to assumption three of the model. In agreement with Taylor [16], it is more realistic

to assume that artillery units would be more concerned about large kill potential weapon systems and further that ground force intelligence would be greater concerned with troop concentrations rather than the individual combatant.

In the model it should be noted that the worth of the forces to the x commander at any time t varied over time up to the assigned values specified at time T; namely, p, q, -r, and -s respectively. This value of forces is indicated by the associated adjoint equations and dual variables. This dynamic combat force potential is more readily demonstrated for the reader in Appendix C in an explanation utilizing a simpler two (2) force model expressed in terms of the Lanchester Square Law. The discussion in Appendix C possesses an intuitive incite to the changing worth of combat units indicating mathematically that a loss of forces to the commander early in the battle is a greater loss than later in the battle, since not only does the commander lose the fighting unit, but the casualty inflicting power of the unit as well.

This initial model further utilizes a fixed kill coefficient which is not easily accepted by tacticians and planners who realize that in reality for a given weapon system the kill rate is primarily a function of the weapons range and lethality at that range, although other conditions like troop training, morale, weather conditions, and many other difficult to model factors are involved. It is this thought that generated Model II.

C. MODEL II.

Now if the assumptions mentioned in Model I remain the same, but the analyst considers data accumulated from past history of war casualties as shown in Fig. 7, extracted from Ref. 18, the first model can be simplified. Since

THEATER OF OPERATIONS	% CASUALTIES	
	SMALL ARMS	MORTAR & ARTILLERY
MEDITERRANEAN	16.8	83.2
EUROPEAN	26.8	73.2

Figure 7. Relative Percent Casualties Produced by Small Arms and Fragmentation Weapon Systems.

approximately 80 percent of the casualties produced are the result of supporting weapon systems, then for the same time specified problem, the casualties inflicted by the primary infantry force can be considered to be insignificant compared to those casualties produced by the supporting secondary system, the field artillery.

Further, if a variable kill coefficient is introduced as a function of lethality, l, and a function of range, r, as presented by Bonder [11], then the objective function remains the same as for Model I, but the state equations are now

$$(15)\quad \dot{x}_1 = - a_2(l,r)\, x_1\, y_2\, \Psi_1$$

(16) $\dot{x}_2 = - a_2(1,r)\ x_2\ y_2\ \Psi_2$

(17) $\dot{x}_3 = - a_2(1,r)\ x_3\ y_2\ \Psi_3$

(18) $\dot{y}_1 = - b_2(1,r)\ y_1\ x_2\ \Phi_1$

(19) $\dot{y}_2 = - b_2(1,r)\ y_2\ x_2\ \Phi_2$

(20) $\dot{y}_3 = - b_2(1,r)\ y_3\ x_2\ \Phi_3$

where $\Phi_3 = 1 - \Phi_1 - \Phi_2$ and $0 \leq \Phi_i, \Psi_i \leq 1$

$\Psi_3 = 1 - \Psi_1 - \Psi_2$

and the Φ_i's are determined for indicated Ψ_i's, and i = 1,2, and 3.

The Hamiltonian becomes

(21) $$H(x,y,p,t;\ \Psi,\ \Phi) = - p_1 a_2(1,r) x_1 y_2 \Psi_1 - p_2 a_2(1,r) x_2 y_2 \Psi_2 - p_3 a_2(1,r) x_3 y_2 \Psi_3 - p_4 b_2(1,r) y_1 x_2 \Phi_1 - p_5 b_2(1,r) y_2 x_2 \Phi_2 - p_6 b_2(1,r) y_3 x_2 \Phi_3$$

and the adjoint system of equations is now

(22) $\dot{p}_1 = p_1 a_2(1,r) y_2 \Psi_1 + p_4$

(23) $\dot{p}_2 = p_2 a_2(1,r) y_2 \Psi_2 + p_4 b_2(1,r) y_1 \Phi_1 + p_5 b_2(1,r) y_2 \Phi_2 + p_6 b_2(1,r) y_3 \Phi_3$

(24) $\dot{p}_3 = p_3 a_2(1,r) y_2 \Psi_3$

(25) $\dot{p}_4 = p_4 b_2(1,r) x_2 \Phi_1$

$$(26)\quad \dot{p}_5 = p_1 a_2(1,r) x_1 \Psi_1 + p_2 a_2(1,r) x_2 \Psi_2 + p_3 a_2(1,r) x_3 \Psi_3 + p_5 b_2(1,r) x_2 \Phi_2$$

$$(27)\quad \dot{p}_6 = p_6 b_2(1,r) x_2 \Phi_3$$

Since the adjoint equations remain dependent on the state variables, the resulting two boundary problem does not readily possess a solution by elementary techniques. The model may still be examined qualitatively.

The model now contains allocation factors of artillery fire dependent upon the size of the opposing forces and the kill capability of that opposing force. This allocation of fire corresponds to the criteria of artillery fire planning presented on priorities and precedence established in Section II-F, where the artillery operations officer, acting rationally, determines what units to engage depending on the forces threat to the accomplishment of the mission. Certainly those targets yielding an immediate threat to the accomplishment of the mission must be engaged, as is the case in the model when the proportion of artillery fire Φ_i, directed at the opposing forces is related to the size of that force and its respective kill capability.

The varying kill potential of the forces dependent on the forces lethality and range can be extracted from figures as in Fig. 8 for the position at a particular instant of time. This weapon system kill coefficient is in consonance with the idea of Weapons Fire Power Potential (WFPP) developed by Technical Operations Incorporated Combat Operations Research Group for U.S. Army Combat Developments Command [18].

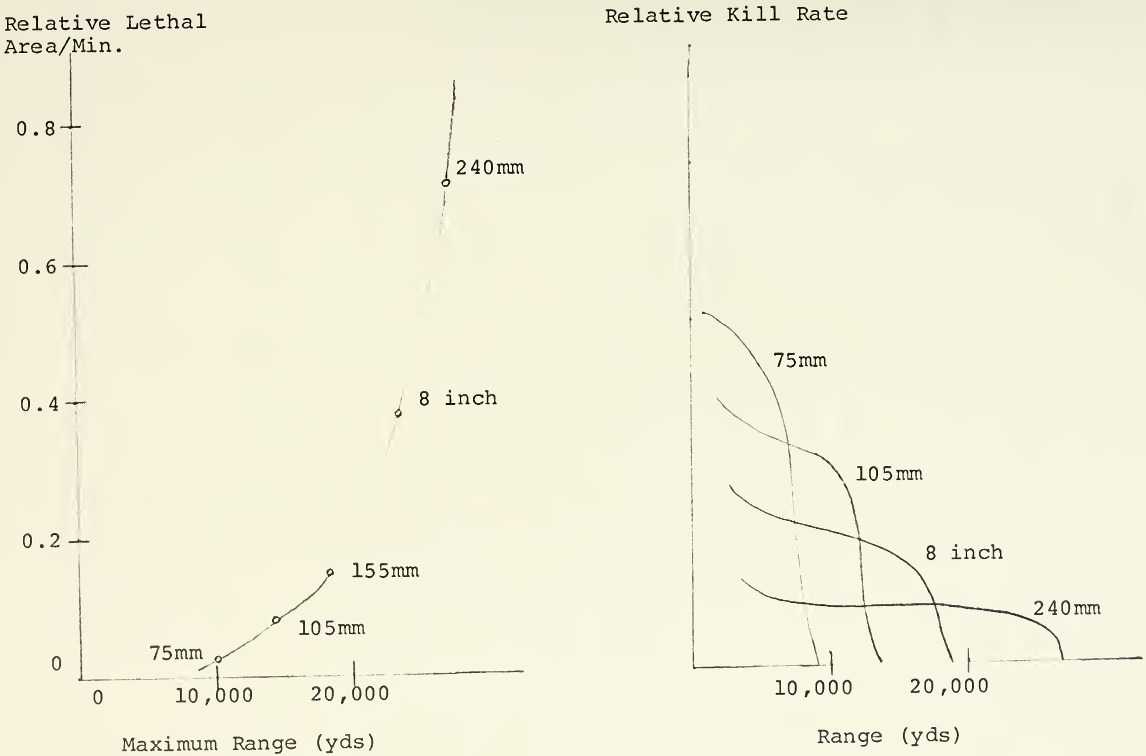


*Figure 8. Artillery Characteristics.

* Extracted from Ref. 4.

WFPP - is defined as the product of the lethality of a projectile delivered by a weapon system times the number of rounds fired during a given period.

Lethality measures were developed for area, point, and anti-armor weapon systems. The parameters used for development of this WFPP included available armament, ammunition types, the tactical situation, vulnerability posture, and target types. Factors excluded from the derivation were intelligence; mobility; command, control, and communications; service support; and morale, esprit de corps, and training. Fire power potential for area type weapons was defined as the product of the mean lethal area of a round and the number of projectiles fired during a given period as provided by the estimate of the expenditure of ammunition. Further the lethality of one round of ammunition for point fire weapons was developed by establishing a relationship between the point fire weapons producing capability and that of area fire weapons employed against personnel. To accomplish this, three assumptions were used. First, it was assumed that the casualty producing potential of a single round of ball ammunition had not changed significantly since World War II (WWII data was used); second, the lethality of a round of ball ammunition was assumed to be a function of its casualty producing capability; and third, the potential of any given round of ball ammunition was assumed to be the same regardless which weapon fired it. With this a feasible relative ranking was established since compatible lethal areas had been developed for both area and point fire family

weapons. Finally combining of WFPP was considered possible since WFPP had been defined as the total maximum lethality brought to bear by the weapon system or systems over a period of time on the target. Actual computation and procedure of the calculation of the WFPP is not included since the reference where the above unclassified information was extracted is classified. The interested reader is directed to Ref. 18.

The kill coefficients as a function of range and lethality may further include in the function a troop movement rate negative or positive to account for retrograde or attack movement of forces, to pertain to other than static situations. Generally, this will further complicate the model and make it more difficult to use an elementary solution technique, but it would make the model more general.

Further, the commanders worth for a given force changing with time, as expressed through the dual variables in the adjoint equations, agrees with reality in the sense that as a battle progresses in fast changing tactical situations, the force commander's value for a given unit surely may vary dependent on the enemy force strength, the mission assigned, or both.

D. GENERAL COMMENTS

Neither model presented included operational losses since the models presented already are complicated enough to solve or analyze already. If the interest is to include operational losses, this can be added to the model using the approach

presented in Ref. 1. The results obtained without the operational loss consideration would be a little optimistic, since the number of survivors without this consideration may be increased.

Excluded also from the models were troop replacement, troop training, and troop morale since these were considered beyond the scope of this paper.

V. SUMMARY AND CONCLUSIONS

A. SUMMARY

The primary interest of this paper was to present an extension to the existing models already developed for target allocation of secondary weapon systems, in particular, the field artillery weapon system as the supporting arms to the infantry maneuver force in the division area. The field artillery system was presented to familiarize the reader with the systems capabilities and limitations, with directed emphasis on the fire planning operations which direct the artillery to the actual target engagement.

A basic one-sided model of the posed division tactical situation of two opposing division size forces consisting of a primary system of three infantry brigades and a secondary system of field artillery, both direct support and general support, was presented using extended Lanchester theory with a constant kill coefficient for the included forces. This model was further extended to use a variable kill coefficient as a function of lethality and range, with both models set up for a specified time, T. Both models resulted in systems of differential equations with solutions not readily obtainable by elementary methods, however the models presented include a more realistic kill coefficient, dependent upon lethality and range.

Further the troop worth to the unit commander was presented through the use of the duals in the adjoint system

of equations. This implies that a soldier lost earlier in the battle was a greater loss to the commander since the soldier and his casualty inflicting power are both loss for a longer time.

B. CONCLUSIONS

The area of target allocation of secondary weapons systems is wide open to expansion and extensions of already developed theoretical models. Particular emphasis is needed in the area of more tractable models closer depicting reality of supporting systems, as the field artillery, to include ammunition constraints, force replacements, troop morale, logistical and transportation interaction, target acquisition, and stochastic behavior. Models considering some of these areas are already appearing in the field, but only in elementary stages. The field artillery, as well as other supporting systems and other associated fields of allocation, has a need for concentrated work in models of this type to help solve the problem of allocation to enhance their contribution in the success of the overall mission.

APPENDIX A

LANCHESTER EQUATIONS

The Lanchester simultaneous differential equations and their solutions which are probably the first attempt at the development of combat models are presented below as a review as extracted from Refs. 12 and 20.

The Square Law is applicable for situations utilizing aimed fire and is developed under the assumption that:

(1) Two forces are engaged in a firefight when each unit or each force is within weapon range of all units of the other force.

(2) Units on each force are homogeneous but the kill rate of the opposing force may be different.

(3) Each firing unit or force is well aware of the location of the opposing units so that when a target is killed, the forces fire is shifted to another target.

(4) Units fire is uniformly distributed over the area in which the surviving forces are located.

The opposing forces are designated as odd and even with the following notation:

x_1, x_2 = number of surviving units (men) on the even or odd force at time t.

x_{10}, x_{12} = initial strength of forces.

p_{21} = single shot probability than an even weapon will kill an odd man.

r_{21} = rate of fire of even weapons against odd units.

The Lanchester equations are:

$$\frac{dx_1}{dt} = - p_{21}\, r_{21}\, x_2 \; ;$$

$$\frac{dx_2}{dt} = - p_{12}\, r_{12}\, x_1$$

The solution was obtained by integrating over time and then equating the results to yield

$$p_{21}r_{21}x_2^2 - p_{12}r_{12}x_1^2 = p_{21}r_{21}x_{20}^2 - p_{12}r_{12}x_{10}^2 \; .$$

The solution obtained is

$$\frac{x_{10}^2 - x_1^2}{x_{20}^2 - x_2^2} = \frac{p_{21}r_{21}}{p_{12}r_{12}}$$

where the ratio of losses squared is indirectly proportional to the effectiveness of the weapons. In situations where the Square Law applies the equations demonstrate that concentration of force is advantageous and it warns against almost certain defeat to the force which commits its forces piecemeal.

The Linear Law is applicable for modeling situations in which forces would use area fire, such as artillery or other fire support systems. The first two assumptions of the Square Law apply again along with the assumptions that

(1) Each firing unit is aware only of the general area in which the opposing units are located and they fire into this area without knowledge of the results of their fire.

(2) Fire from survivor units is uniformly distributed over the area in which the opposing forces are located.

The probability of opposing units killing each other is now a function of areas in the Linear Law. The following notation is listed:

A_1, A_2 = areas in which odd and even forces are located.

ae_1, ae_2 = area of effectiveness (man) of a single shot.

r_1, r_2 = rate of fire for an odd or even man.

$PA_{12} = \frac{r_1 ae_1}{A_2}$ effectiveness coefficient of even against odd.

The Lanchester equations for area fire then are

$$\frac{dx_1}{dt} = -p_{A_{21}} x_2 x_1$$

$$\frac{dx_2}{dt} = -p_{A_{12}} x_2 x_1$$

The solution obtained by integrating over time and equating results yield

$$p_{A_{21}} - p_{A_{12}} = p_{A_{21}} x_{20} - p_{A_{12}} x_{10}$$

From this solution it is obvious that

$$\frac{x_{20} - x_2}{x_{10} - x_1} = \frac{p_{A_{12}}}{p_{A_{21}}}$$

The ratio of losses under the Linear Law are indirectly proportional to the weapons effectiveness coefficients and there is no advantage to be gained by concentrating forces.

The solutions of the Lanchester equations represent average values. The equations are an example of a

deterministic model in that the outcome (rate of attrition) directly result from specified force sizes and kill coefficients which are the initial conditions.

Lanchester's Square Law and Linear Law can be modified by the addition of probability theory. The equations can be applied to areas where theoretical and statistical investigations can lead to useful results. However, the analyst must bear the limitations of the original equations, when they are put to use.

The equations depart from reality in the areas listed below. Many extensions have already been presented by relaxing the assumptions used in the original equations.

(1) Each unit is within range of all enemy units and that kill probability is not a function of range but it is constant.

(2) All units on both forces are homogeneous and therefore are not considering the fact that opposing units may consist of infantry, artillery, armor, tactical air, naval forces, etc.

(3) No provision is made to incorporate intensities of combat into the situation.

(4) There is no allocation for tactical decisions to be made by the commanders for employment of forces.

(5) Random events of probability theory are excluded from the existing equations.

(6) The attrition coefficients do not vary with time.

APPENDIX B

EXAMPLE OF MODEL USING LANCHESTER EQUATIONS

One of the ways in which the Lanchester equations depart from reality was item (4) as in Appendix A which states that there was no provision for tactical decisions by the commander. Weiss [21] did in fact use a valid analysis in his model that provided for decisions by the commander. The tactic that Weiss has analyzed is the selection of targets to be attacked. This approach is incorporated in his model which includes two forces in contact where each force consists of a primary system and a secondary system. The primary system is maneuver forces such as infantry or armor and the secondary system can be thought of as field artillery or tactical air.

The battle as a function of time can be depicted by Fig. 9.

The Lanchester equations are

$$\frac{dx_1}{dt} = -k_{21}x_2 - \Psi k_{41}x_4$$

$$\frac{dx_2}{dt} = -k_{12}x_1 - \Phi k_{32}x_3$$

$$\frac{dx_3}{dt} = -(1-\Psi)\ k_{43}x_4$$

$$\frac{dx_4}{dt} = -(1-\Phi)\ k_{34}x_3$$

where

x_1, x_2 = the number of primary weapons for odd or even at time t.

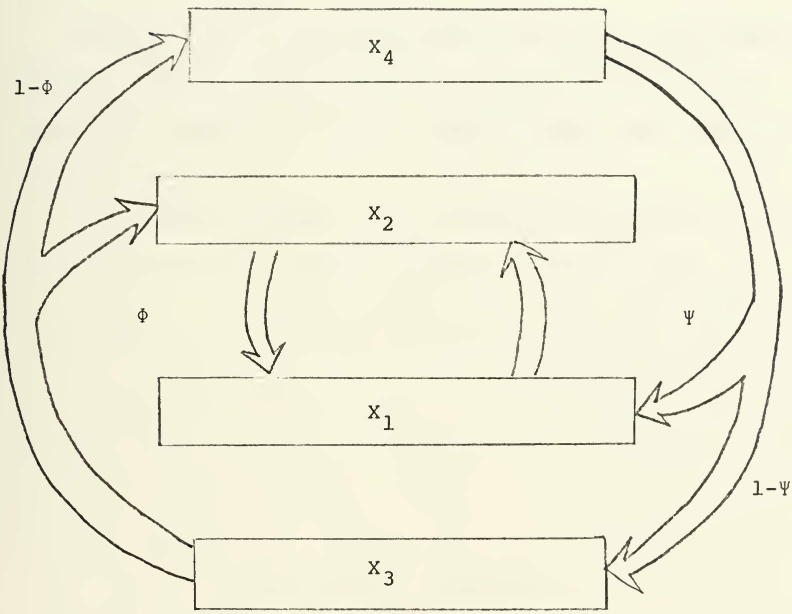


Figure 9. Primary and Secondary Weapon Systems

x_3, x_4 = the number of secondary weapons (artillery or air) for odd or even at time t.

k_{ij} = rate at which the ith weapon can kill the jth weapon.

Ψ, Φ = the fraction of surviving units of the 4,3 type directed at weapons systems of the 1,2 type.

Optimum tactics are determined, using the above model of Weiss's, and the effects on force composition are also determined depending upon the weapon range, cost, and performance parameters.

The problem becomes one of choosing the proper (Ψ, Φ) as a function of time. Weiss further stipulates that $k_{21} = k_{12} = 0$. Here Weiss is stating that each sides primary system is overshadowed by a powerful secondary system to the point where the primary system is insignificant. This implies that the outcome of the engagement will be decided by the secondary system before the effects of the primary system becomes significant. Weiss solved this problem by assuming that Ψ and Φ, the allocation variables, would only take on the values of 0 and 1 and they would remain constant until termination of the battle.

Weiss further considered the value of supporting weapon system in the paper. This implies that the weapon system is one that has an area of effectiveness rather than point effectiveness. However, the Lanchester equations used in the model are those of the <u>Square Law</u> or aimed fire. If the

Linear Law had been used the model would have been too complex for solution. The model indicates results that may be examined in greater detail by a working model in the form of a computer simulation.

APPENDIX C

DYNAMIC COMBAT POTENTIAL

The worth of combat units in dynamic combat situations can be vividly demonstrated through the use of a simple combat model using the Lanchester Square Law discussed in Appendix A.

Consider a combat situation consisting of two opposing forces X and Y each inflicting casualties upon the other with kill coefficients b and a respectively.

The state equations for this system then are

$$(1c) \qquad \frac{dx}{dt} = -\,a\,y$$

$$(2c) \qquad \frac{dy}{dt} = -\,b\,x$$

and the adjoint system equations are

$$(3c) \qquad \frac{dp_1}{dt} = b\,p_2$$

$$(4c) \qquad \frac{dp_2}{dt} = a\,p_1$$

If we use the vector notation below then equations 1c, 2c, 3c, and 4c become

$$(5c) \qquad \frac{d\vec{X}}{dt} = \begin{pmatrix} 0 & -a \\ -b & 0 \end{pmatrix} \vec{X} \qquad\qquad (6c) \qquad \frac{d\vec{P}}{dt} = \begin{pmatrix} 0 & b \\ a & 0 \end{pmatrix} \vec{P}$$

$$(7c) \qquad p_1 \frac{dx}{dt} + p_2 \frac{dy}{dt} = p_1(-ay) + p_2(-bx)$$

$$(8c) \qquad x \frac{dp_1}{dt} + y \frac{dp_2}{dt} = x\,(bp_2) + y\,(ap_1)$$

hence by combining (7c) and (8c)

$$(9c) \quad p_1 \frac{dy}{dt} + p_2 \frac{dy}{dt} + x \frac{dp_1}{dt} + y \frac{dp_2}{dt} = 0 = \frac{d}{dt}(xp_1 + y\, p_2)$$

or $\frac{d}{dt} (\vec{X} \cdot \vec{P}) = 0$

which implies that $\vec{X} \cdot \vec{P}$ = constant,

or $p_1(T) = A$ (some constant) and $p_2(T) = B$ (some constant).

Or

$$(10c) \quad x(t)\, p_1(t) + y(t)\, p_2(t) = x(T)\, p_1(T) + y(T)\, p_2(T) = V,$$

where V represents the value of the engagement, and p_1 and p_2, the dual variables (tangential variables) are

$$(11c) \quad p_1 = \frac{\partial V}{\partial x}$$

$$(12c) \quad p_2 = \frac{\partial V}{\partial y}$$, the gradients of value of the engagement.

But this is the same approach used by G. Bliss (1876-1951) [3, 13] in developing range tables for correcting artillery fire due to abnormal air densities, weights of projectiles, winds and other factors shortly after World War I. The p's (dual variables) may be thought of as the tangential coordinates (line coordinates) of the trajectory of the battle represented by equations 1c and 2c, that is, $x = x(t)$ and $y = y(t)$, the solution of 1c and 2c defines a curve in the x,y plane. The <u>duality</u> of Euclidean geometry (after adding the ideal point at infinity) states that it is possible

to represent a curve either as a sequence of points (point coordinates) or and envelop of tangents (line coordinates), when the points are transformed by the transposed (dual) matrix of transformation. It should be noted that a linear differential equation can be considered as the limit of the linear equation.

The condition $\vec{X} \cdot \vec{P}$ = constant has an interesting and important interpretation. Consider a battle lasting a specified length of time, T. Hence

$$\text{(11c)} \quad x(t)\, p_1(t) + y(t)\, p_2(t) = x(T)\, p_1(T) + y(T)\, p_2(T)$$

as in equation 10c.

If at time t the x commander had $\Delta x(t)$ less troops, then this would cause the x commander to have less troops at the end of the battle and the y force to have more. In fact, the p's (duals) tell us how much, which can be seen below:

$$\text{(12c)} \quad [x(t) - \Delta x(t)]p_1(t) + y(t)\, p_2(t) = [x(T) - \Delta x(T)]p_1(T) + [y(T) + \Delta y(T)]p_2(T)$$

combining 11c and 12c

$$\text{(13c)} \quad \Delta x(t)\, p_1(t) = \Delta x(T)\, p_1(T) - \Delta y(T)\, p_2(T)$$

if $p_1(T) = 1$ and $p_2(T) = -1$, it is obvious why the p's are referenced as the value of the forces. Equation 13c then becomes,

$$\text{(14c)} \quad \Delta x(t)\, p_1(t) = \Delta x(T) + \Delta y(T)$$

The variable $p_1(t)$ indicates the effect of the loss of one

X soldier at time t would have on the outcome of the battle. Expressing the value of the engagement, V, in terms of surviving soldiers, it becomes apparent that

$$p_1(t) = \frac{\partial V}{\partial x}(t) \quad \text{and} \quad p_2(t) = \frac{\partial V}{\partial x}(t) \; .$$

Bliss's idea for the development of air density corrections for the artillery range tables was similar.

The underlying methematical structure used here (duality) manifests itself in many of the modern operations research optimization tools. Recall that it was shown for

$$\frac{d\vec{X}}{dt} = A\vec{X} \quad \text{and} \quad \frac{d\vec{P}}{dt} = - A\vec{P}$$

that $\vec{X} \cdot \vec{P}$ = constant. The finite dimensional analogue of this relationship is for

$$A\vec{x} = \vec{b} \quad \text{and} \quad A^T \vec{y} \geq c$$

where the following must occur,

$$\vec{y} \cdot \vec{b} = \vec{c} \cdot \vec{x}.$$

When this is extended to non-negative variables for

$$A\vec{x} = \vec{b} \quad \text{and} \quad A^T\vec{y} \geq c \quad \text{with} \quad \vec{x} \geq 0,$$

then the following must be true

$$\vec{y} \cdot \vec{b} \geq \vec{c} \cdot \vec{x}$$

The latter relationships may then be developed to yield many Linear Programming theory results. For example, an immediate consequence is that for $\vec{x}$ that maximizes $c \cdot \vec{x}$ subject to

$$A \vec{x} = \vec{b} \quad \text{and} \quad \vec{x} \geq 0,$$

a sufficient condition is

$$A^T (B^{-1})^T C_B - C \geq 0,$$

which in Linear Programming literature is expressed,

$$Z_j - C_j \geq 0.$$

Therefore in summarizing the discussion the reader should note the importance of the dual variables as discussed to yield an intuitive feeling for the worth of the combat units in a dynamic situation. Certainly this is the case in the real world, that a loss of troops early in the battle (assuming no replacements are allowed) is a greater loss to the commander since that soldier is lost for a greater length of time. Besides the loss of the soldier there is an added loss of the casualty inflicting power that soldier would have yielded over the remainder of the battle. It was further shown that his theory of duality used here further manifests itself in the area of Linear Programming.

APPENDIX D

STANDARD TACTICAL MISSIONS

Artillery with a tactical mission of--	Answers calls for fire in priority from--	Establishes liaison with--	Establishes communication with--	Has as its zone of fire--	Furnishes forward observers--	Is positioned by--	Has its fires planned by--
General support.	1. Force artillery headquarters. 2. Own observers.	No inherent requirement.	No inherent requirement.	Zone of supported unit formation.	No inherent requirement.	Force artillery headquarters.	Force artillery headquarters.
General support-reinforcing.	1. Force artillery headquarters. 2. Reinforced artillery unit. 3. Own observers.	Reinforced artillery unit.	Reinforced artillery unit.	Zone of supported unit/formation to include zone of reinforced artillery unit.	Upon request of reinforced artillery unit, subject to prior approval of force artillery headquarters	Force artillery headquarters or, subject to prior approval, the reinforced artillery unit.	Force artillery headquarters.
Reinforcing.	1. Reinforced artillery unit. 2. Own observers. 3. Force artillery headquarters.	Reinforced artillery unit.	Reinforced artillery unit.	Zone of fire of reinforced artillery unit.	Upon request of reinforced artillery unit.	Reinforced artillery unit or ordered by force artillery headquarters.	Reinforced artillery unit.
Direct support.	1. Supported unit. 2. Own observers. 3. Force artillery headquarters.	Supported unit (down to battalion level).	Supported unit.	Zone of supported unit.	To (each*) company-size maneuver element of supported unit.	Unit commander, as deemed necessary, or ordered by force artillery headquarters.	Develops own fire plan.

*each applicable to US only.

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13. ABSTRACT

Several models of the problem of target selection for field artillery fire as a supporting weapon system to a maneuver element in a division field environment are presented in this thesis. The field artillery system, its capabilities and limitations, as well as, the criteria utilized by military decision makers to provide timely, accurate, and effective artillery fire support to the maneuver commander, is covered to familiarize the analyst with the system to be modeled. A differential equation model using Lanchester Theory of Combat and the mathematical technique of optimal control to the target allocation problem is presented. A second model presented uses an allocation of fire dependent upon the kill potential and capability of the respective forces. The kill potential varies with the lethality and range of the weapon system from that force. A discussion of the worth of combat units in dynamic combat situations is also presented. The conclusion reached is that there is a dire need for more models in the area of target allocation that can clearly depict reality and still maintain a certain mathematical tractability.

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